

Transient Stability Analysis of 330kV Nigerian Transmission Network, using Eigenvalue Principles

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ABSTRACT: Power systems networks are subject to disturbances during operation due to the interconnected nature of the system. These disturbances include load variations, transformer tap-changing action, line outages due to various types of faults of which earth fault is the most common, etc. To better understand these conditions such that adequate control measures can be implemented, the power system network is modeled as a single-phase representing the three phases for a balanced condition analysis. A survey of already existing literature shows that most work done on power system stability majorly focused on the generators and the buses at which they are connected. In this article we present transmission network stability analysis using eigenvalue calculated from the system linearized eigenvectors. The node admittance parameters computed from the line reactance were applied to the eigenvalue-eigenvector model to determine the system stability. The 7x7 nodal admittance matrix formed, were fed to eigenvalue-eigenvector model at Matlab Command prompt to compute the right and left eigenvectors and the diagonal or eigenvalue of the network under steady-state and faulty condition.

KEYWORDS: Transmission Network, Transient Stability, Eigenvalue Analysis, Eigenvector

I. INTRODUCTION

The nature of the power system is such that it is subject to perturbations during its operations. Large disturbances occur when the system is subjected to a severe fault which may be due to sudden loss of generator, line outages from the system, or heavy short circuits. Small perturbations arise due to load variations and equipment switching such as tap-changing transformer actions. In any case the power system experiences some kind of instability; transient or steady-state of which both are of importance to power system engineers. The interconnected nature of the transmission network makes it possible for the transient associated with it to decay very rapidly. Thus, network transients will have died out by the time the solution of the swing equation is solved by a one-time step [1]. Hence, it is adequate to regard the network during the electromechanical transient conditions, as though it were passing directly from one steady-state to another. The fundamental frequency (60 or 50 Hz) variations are assumed to be micro-processes, such that only the variations in the envelopes (amplitude modulation) of current and voltage waveforms are considered for stability analysis.

Historically the issue of generator dynamic instability or frequency oscillation associated with large power system networks dates back to the late 1950s and early 1960s when generators were built with automatic voltage regulators [2]. The generators swing either in local or inter-area mode with frequencies ranging from 0.1 to 2 Hz. Since then various methods of stabilizing the generator oscillations such as power system stabilizer (PSS), Static Var Compensators, etc, had been extensively studied [3]. [4] Used eigenvectors corresponding to the power system oscillatory modes to determine the relative motions of the machines at this mode and to figure out which machines were swinging with and against each other. This information enabled them to determine which machine is the candidate for PSS placement. Rodney presented Eigenvalue Tracking (ET) method in [5]. He carried out a comprehensive comparative report between ET, Participation Factor (PF), Residual (RES) etc., and methods of determining PSS placement among the generators in order to obtain stability. His work noted that ET methods showed better performance than other methods compared.

Bulge chasing algorithm called the BR algorithm which is a novel and efficient method to find all Eigenvalues of upper Heisenberg matrices was introduced by [6]. In his work, he analyzed the differences between the BR and the QR algorithms with performance comparison in terms of CPU time based on stopping criteria and storage requirement based on Eigenvalue calculation. He noted that the BR algorithm is a better choice for small signal stability studies when compared to QR algorithms which incorporate matrix sparsity in its routine. Modal or Eigenvalue analysis of small-signal power system stability was rapidly analyzed and presented in [7]. He used the PSSTME Version 31

software to analyze the instability problem of an inter-area mode swing and suggested PSS location based on the result of the analysis. He noted that the use of High Voltage Direct Current (HVDC) and Flexible Alternating Current Transmission System (FACTS) improves the system dynamics and stability with their ability to change the power flow directly and indirectly within millisecond (ms). The effect of small disturbances on system stability was investigated in [8] to determine the best position for power system stabilizer, to restore system stability by evaluating the eigenvalues and eigenvectors of the linearized differential and algebraic equation of the system.

A survey of already existing literature shows that most work done on power system stability majorly focused on the generators and the buses at which they are connected. In this article we present transmission network stability analysis using eigenvalue calculated from the system linearized eigenvectors. For the analysis of balanced conditions, a single-phase representation of the three-phases is used. Without the balanced steady-state representation of the transmission network, stability analysis of large practical power systems would be impractical. For complex transient stability analysis involving generators, stators, and network transients, electromagnetic transient programs (EMTP) are used [9]. For conventional transient stability analysis, the network representation is similar to that for power-flow analysis. The most convenient form of network representation is in terms of the node admittance matrix.

II. TECHNICAL CONSIDERATION

In modeling loads; dynamic loads are represented as induction and synchronous motors, and their treatment is similar to that of synchronous machines. Static loads are represented as part of the network equations. Loads with constant impedance characteristics are the simplest to handle and are included in the node admittance matrix. [1] Nonlinear loads are modeled as an exponential or polynomial function of bus voltage magnitude and frequency. The net effect is that a nonlinear static load model is treated as a current injection at the appropriate node in the network equation. Figure 1 shows a typical power system in modular form.

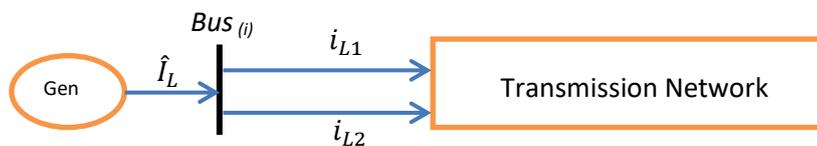


Figure 1: Sample Nodal Network in modular form.

The value of the node current from ground into the network is

$$\hat{I}_L = i_{L1} + i_{L2} = -\frac{P_L - jQ_L}{\hat{V}_L^*} \tag{1}$$

Where \hat{V}_L^* is the conjugate of the load bus voltage, P_L and Q_L are portions of the active and reactive components of the load of which vary as nonlinear functions of V_L and Frequency deviation Δf . For an inductive load, Q_L is positive.

The overall network/load representation comprises a large sparse nodal admittance matrix equation with a structure similar to that of the power-flow problem. The network equation is written in matrix form as:

$$\hat{I}_L = Y_N \hat{V} \tag{2}$$

The node admittance matrix Y_N is symmetrical, except for dissymmetry introduced by phase-shifting transformers. Within the time frame of transient stability simulations, transformer taps and phase-shift angles do not change. Therefore, the elements of the matrix remain constant except for change introduced by network switching operations or external loads. The effect of generators, nonlinear static loads, dynamic loads, and other devices such as HVDC converters and regulated VAR compensators are reflected as boundary conditions providing additional relationships between \hat{V} and \hat{I} at the respective nodes. In contrast to power-flow analysis, tie line power-flow control, limits on generator reactive power output and the slack bus make-up for the unknown losses and need not be considered in transient stability analysis [1].



For (i^{th}) number of buses, Y_{bus} is expressed as

$$\frac{1}{Z_{bus}} = \frac{1}{r \pm jx} = Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & \dots & Y_{1i} \\ Y_{21} & Y_{22} & \dots & Y_{2i} \\ Y_{31} & Y_{32} & \dots & Y_{3i} \\ \vdots & \vdots & \dots & \vdots \\ Y_{i1} & Y_{i2} & \dots & Y_{ii} \end{bmatrix} \quad (3)$$

The off diagonal elements of the matrix are called Mutual Admittances while the diagonal elements are called Self Admittances. $r \pm jx$ Represent the branch resistances and reactance respectively. Equation (4) defines the Mutual admittance and expressed as the negative of the admittance of the line between buses 1 and i while equation (5) defines the Self admittance as the sum of all the admittances connected to that bus.

$$Y_{i1} = Y_{1i} = -y_{i1} = -y_{1i} \quad (4)$$

$$Y_{ii} = \sum_{\substack{i=0 \\ i \neq 1}}^n y_{ii} \quad (5)$$

III. EIGENVALUE ANALYSIS

Equation (2) can be represented in steady state-space form as follows:

$$Ax = \hat{x} \quad (6)$$

To obtain the solution of equation (6), a scalar parameter λ called the eigenvalue is introduced such that equation (6) becomes;

$$Ax = \lambda x \quad (7)$$

Where $A = [a_{ij}]$ an $n \times n$ square matrix is x is an $n \times 1$ vector and λ is a number (scalar) parameter [1, 10]. Clearly the solution $x = 0$ for λ is usually not useful and thus is neglected.

For non-trivial solutions i.e. $x \neq 0$, the values of λ are called the eigenvalues and the characteristics values or latent roots of the matrix A and the corresponding solutions of equation (7) are called eigenvectors or characteristic vectors of A . Expressed as separate equations we have;

$$A \cdot x - \lambda x = 0 \quad (8)$$

$$(A - \lambda I)x = 0 \quad (9)$$

Notice that the unit matrix I was introduced so that λ can be subtracted from matrix A . Now for equation (9) to have a non-trial solution, determinant of $|A - \lambda I|$ must be equal to zero. Hence

$$|A - \lambda I| = \begin{vmatrix} (a_{11} - \lambda) & a_{12} & \dots & a_{1i} \\ a_{21} & (a_{22} - \lambda) & \dots & a_{2i} \\ \vdots & \vdots & \dots & \vdots \\ a_{i1} & a_{i2} & \dots & (a_{ii} - \lambda) \end{vmatrix} = 0 \quad (10)$$

Expansion of equation (10) gives the characteristics equation. The n solutions of $\lambda = \lambda_1, \lambda_2, \lambda_3$

.... λ_n are eigenvalues of A .



Writing equation (11) in determinant form gives;

$$|Y_N - \lambda I| = \begin{vmatrix} (y_{11} - \lambda) & y_{12} & \dots & y_{1i} \\ y_{21} & (y_{22} - \lambda) & \dots & y_{2i} \\ \vdots & \vdots & \dots & \vdots \\ y_{i1} & y_{i2} & \dots & (y_{ii} - \lambda) \end{vmatrix} = 0 \tag{12}$$

The network admittance Y_N can be obtained as follows:

$$y_{11}(\text{Benin}) = \sum[2(y_{12}) + y_{13}] = 4.6713 \tag{13}$$

$$y_{22}(\text{Ajaokuta}) = \sum[2(y_{21}) + 2(y_{24}) + 2(y_{25})] = 4.8110 \tag{14}$$

$$y_{33}(\text{Ihebor}) = \sum[y_{31} + 0 + 0 + \dots] = 3.0211 \tag{15}$$

$$y_{44}(\text{Geregu}) = \sum[2(y_{42}) + 0 + 0 + \dots] = 1.5106 \tag{16}$$

$$y_{55}(\text{Lokoja}) = \sum[2(y_{52}) + 2(y_{56}) + y_{57}] = 6.6007 \tag{17}$$

$$y_{66}(\text{Eastmain}) = \sum[2(y_{65}) + 0 + 0 + \dots] = 1.6502 \tag{18}$$

$$y_{77}(\text{Markudi}) = \sum[y_{75} + 0 + 0 + \dots] = 3.3003 \tag{19}$$

$$Y_N = \begin{bmatrix} y_{11} & y_{12} & y_{13} & y_{14} & y_{15} & y_{16} & y_{17} \\ y_{21} & y_{22} & y_{23} & y_{24} & y_{25} & y_{26} & y_{27} \\ y_{31} & y_{32} & y_{33} & y_{34} & y_{35} & y_{36} & y_{37} \\ y_{41} & y_{42} & y_{43} & y_{44} & y_{45} & y_{46} & y_{47} \\ y_{51} & y_{52} & y_{53} & y_{54} & y_{55} & y_{56} & y_{57} \\ y_{61} & y_{62} & y_{63} & y_{64} & y_{65} & y_{66} & y_{67} \\ y_{71} & y_{72} & y_{73} & y_{74} & y_{75} & y_{76} & y_{77} \end{bmatrix} \tag{20}$$

Therefore,

$$Y_N = \begin{bmatrix} 4.6713 & -1.6502 & -3.0211 & 0 & 0 & 0 & 0 \\ -1.6502 & 4.8110 & 0 & -1.5106 & -1.6502 & 0 & 0 \\ -3.0211 & 0 & 3.0211 & 0 & 0 & 0 & 0 \\ 0 & -1.5106 & 0 & 1.5106 & 0 & 0 & 0 \\ 0 & -1.6502 & 0 & 0 & 6.6007 & -1.6502 & -3.3003 \\ 0 & 0 & 0 & 0 & -1.6502 & 1.6502 & 0 \\ 0 & 0 & 0 & 0 & -3.3003 & 0 & 3.3003 \end{bmatrix} \tag{21}$$

The 7 x 7 Matrix of equation 21 was coded into Matlab workplace command prompt and the right eigenvector ($V = \hat{I}_L$), the eigenvalues ($D = \lambda$) and the left eigenvector ($W = \hat{V}$) was computed and results are presented below.

V. RESULT AND ANALYSIS

The term right eigenvector (V), eigenvalues (D), and left eigenvector (W) are built in variables which Matlab assigns vector matrices upon computation. These terms are synonymous to the variables used in our analysis as represented.

$$V = \hat{I}_L =$$

$$0.2274 \quad -0.7100 \quad 0.2282 \quad 0.0619 \quad -0.3780 \quad -0.3834 \quad 0.1391$$



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$$\begin{matrix}
 -0.4050 & 0.2916 & 0.7387 & 0.1301 & -0.3780 & 0.0702 & -0.1420 \\
 -0.1129 & 0.4873 & -0.4069 & 0.0460 & -0.3780 & -0.5269 & 0.2357 \\
 0.0805 & -0.0745 & -0.3482 & 0.0770 & -0.3780 & 0.1542 & -0.7853 \\
 0.7538 & 0.3166 & 0.1332 & 0.3292 & -0.3780 & 0.4121 & 0.2723 \\
 0.1156 & 0.0576 & 0.0345 & 0.8945 & -0.3780 & 0.2750 & 0.1556 \\
 -0.4283 & -0.2535 & -0.3106 & 0.2502 & -0.3780 & 0.5488 & 0.4357
 \end{matrix}$$

(22)

$D = \lambda =$

$$\begin{matrix}
 9.1093 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 7.4227 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 4.7157 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & -1.0428 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0.0000 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0.8224 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 1.2375
 \end{matrix}$$

(23)

$W = \hat{V} =$

$$\begin{matrix}
 0.2274 & -0.7100 & 0.2282 & -0.0619 & -0.3780 & -0.3834 & 0.1391 \\
 -0.4050 & 0.2916 & 0.7387 & -0.1301 & -0.3780 & 0.0702 & -0.1420 \\
 -0.1129 & 0.4873 & -0.4069 & -0.0460 & -0.3780 & -0.5269 & 0.2357 \\
 0.0805 & -0.0745 & -0.3482 & -0.0770 & -0.3780 & 0.1542 & -0.7853 \\
 0.7538 & 0.3166 & 0.1332 & -0.3292 & -0.3780 & 0.4121 & 0.2723 \\
 -0.1156 & -0.0576 & -0.0345 & 0.8945 & 0.3780 & -0.2750 & -0.1556 \\
 -0.4283 & -0.2535 & -0.3106 & -0.2502 & -0.3780 & 0.5488 & 0.4357
 \end{matrix}$$

(24)

After the steady-state simulation, a three phase line to earth (ground) fault was applied to one of the Ajaokuta-Lokoja axis and the line was tripped by a circuit breaker at exactly 0.5 sec; figure 3. The resultant admittance at bus 2 (Ajaokuta) at this instance became,

$$Y_{22} = 3.3003 + 1.5106 + 1.6502 = 6.4611 \tag{25}$$

The changes in the total reactances of all branches connected to bus 2 was due to the fact that the faulted line’s admittance reduced to zero by virtue of equation 2 since the voltage between the faulted line and earth become zero at

that instance. During the period the fault subsist, the parallel circuit in that axis became single circuit whose effect was to increase the resultant reactance at the affected node.

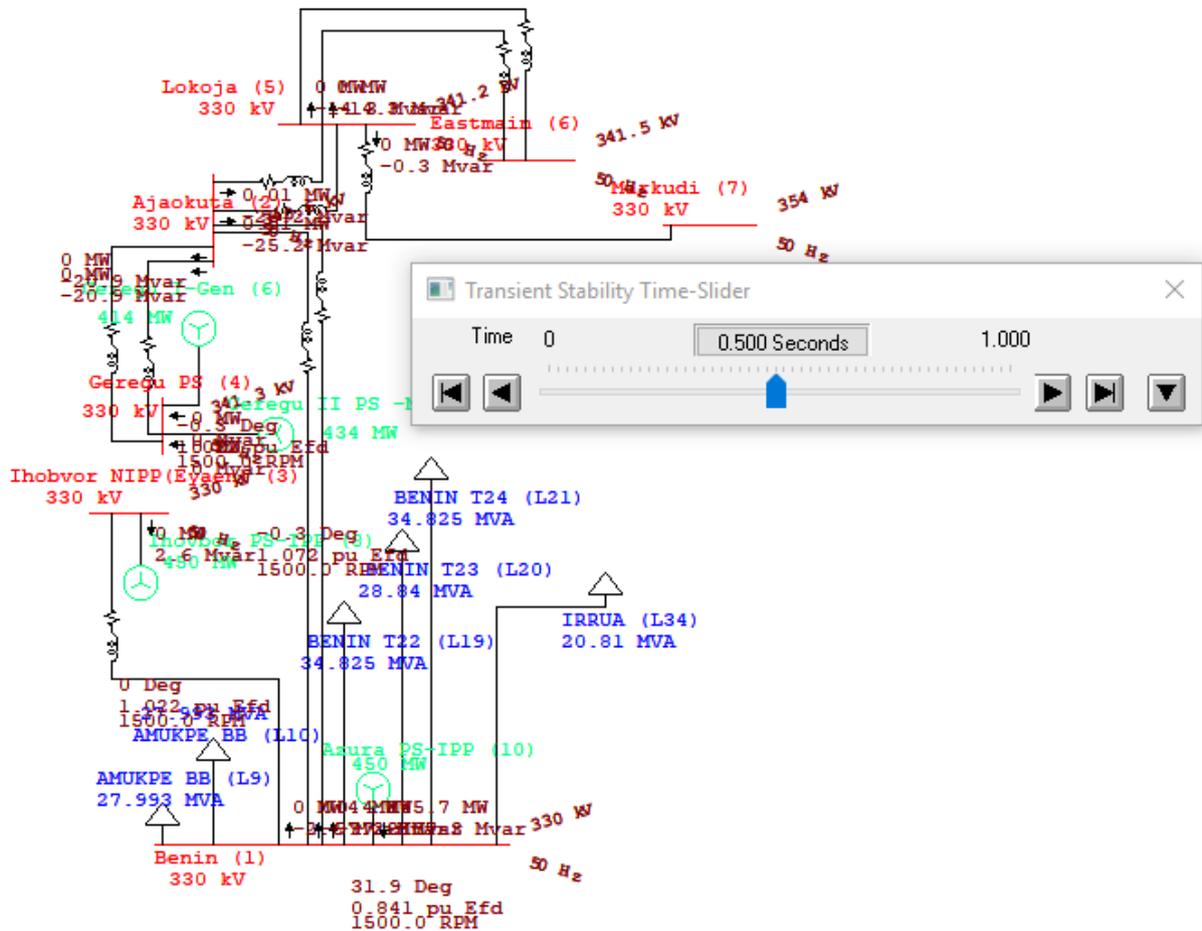


Figure 3: Middle Belt Grid simulated under faulty condition.

This affected the entire system admittance Y_{Nf} and the eigenvalue, eigenvector results as follows, where the subscript (f) included in the variables indicates new parameters under faulty condition.

$$Y_{Nf} = \begin{bmatrix} 4.6713 & -1.6502 & -3.0211 & 0 & 0 & 0 & 0 \\ -3.3003 & 6.4611 & 0 & -1.5106 & -1.6502 & 0 & 0 \\ -3.0211 & 0 & 3.0211 & 0 & 0 & 0 & 0 \\ 0 & -1.5106 & 0 & 1.5106 & 0 & 0 & 0 \\ 0 & -1.6502 & 0 & 0 & 6.6007 & -1.6502 & -3.3003 \\ 0 & 0 & 0 & 0 & -1.6502 & 1.6502 & 0 \\ 0 & 0 & 0 & 0 & -3.3003 & 0 & 3.3003 \end{bmatrix} \quad (26)$$

$$V = \hat{I}_{Lf} =$$

$$\begin{bmatrix} 0.2820 & -0.4230 & 0.3173 & 0.0494 & -0.3780 & 0.1085 & -0.2182 \\ -0.6403 & 0.3310 & 0.7065 & 0.1047 & -0.3780 & -0.1258 & 0.0561 \\ -0.1262 & 0.2645 & -0.4409 & 0.0366 & -0.3780 & 0.1900 & -0.3057 \\ 0.1171 & -0.0789 & -0.2897 & 0.0615 & -0.3780 & -0.8829 & 0.1311 \end{bmatrix}$$



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$$\begin{matrix}
 0.6127 & 0.6390 & 0.1726 & 0.3224 & -0.3780 & 0.1974 & 0.5067 \\
 0.0885 & 0.1110 & 0.0416 & 0.9045 & -0.3780 & 0.1106 & 0.3326 \\
 -0.3125 & -0.4633 & -0.3006 & 0.2439 & -0.3780 & 0.3249 & 0.6865
 \end{matrix} \tag{27}$$

$$D = \lambda_f =$$

$$\begin{matrix}
 9.7702 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 7.8521 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 5.1951 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & -1.0620 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & -0.0000 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 1.2954 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0.8641
 \end{matrix} \tag{28}$$

$$W = \hat{V}_f =$$

$$\begin{matrix}
 0.4973 & -0.6401 & 0.4622 & -0.0983 & -0.5547 & 0.2030 & -0.3659 \\
 -0.5645 & 0.2505 & 0.5146 & -0.1042 & -0.2774 & -0.1176 & 0.0470 \\
 -0.2226 & 0.4003 & -0.6422 & -0.0727 & -0.5547 & 0.3553 & -0.5124 \\
 0.1032 & -0.0597 & -0.2110 & -0.0612 & -0.2774 & -0.8256 & 0.1099 \\
 0.5402 & 0.4835 & 0.1257 & -0.3206 & -0.2774 & 0.1846 & 0.4248 \\
 -0.0781 & -0.0840 & -0.0303 & 0.8994 & 0.2774 & -0.1034 & -0.2788 \\
 -0.2756 & -0.3506 & -0.2189 & -0.2425 & -0.2774 & 0.3038 & 0.5754
 \end{matrix} \tag{29}$$

Under steady-state, the system is found to have real eigenvalues probably due to the approximation introduced during line admittance formulation and computation. The real eigenvalues obtained are positives indicating an aperiodic instability except at bus 4 (Geregu) where the network is found to have a negative real eigenvalue which indicates a decaying condition or mode. When fault was applied, the same bus 4 (Geregu) recorded a negative real eigenvalue but with a slightly lower value. This entails a slower transient decay constant. Also at bus 5 (Lokoja) a higher value of -0.0001 negative real eigenvalue was recorded due to the faulty condition. However this mode decays faster as shown in figure 3 where the fault was cleared after 1 second. The rest of the buses are characterized by aperiodic transient which ranges from bus 1 down to bus 7 in that order.

VI.CONCLUSION

The transient stability analysis of Nigerian 330 kV Middle Belt Grid had been analyzed in this article with regard to the eigenvalue characteristics of the network. The results presented showed regions of the network under aperiodic instability and decay mode during the faulty condition and steady-state.

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